Lecture Series on

Intelligent Control

Lecture 19

General Fuzzy Systems

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General Fuzzy Systems

Multiple Input Multiple Output Fuzzy Systems

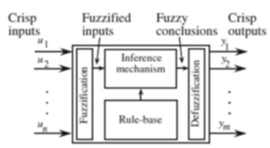

Figure 5.20: Fuzzy system (controller).

MIMO fuzzy system = *m* MISO fuzzy systems

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Fuzzy Systems:

- **Standard fuzzy systems**
- **Functional fuzzy systems**

Consequents are not linguistic, but a function:

$$b_i = g_i(\cdot) = a_{i,0} + a_{i,1}(u_1)^2 + \cdots + a_{i,n}(u_n)^2$$

or

$$b_i = g_i(\cdot) = \exp\left[a_{i,1} sin(u_1) + \cdots + a_{i,n} sin(u_n)\right]$$

Let R denote the number of rules. For the functional fuzzy system we can use an appropriate operation for representing the premise (e.g., minimum or product), and defuzzification may be obtained using

$$y = \frac{\sum_{i=1}^{R} b_i \mu_i(z)}{\sum_{i=1}^{R} \mu_i(z)} \qquad (5.3)$$

where $\mu_i(z)$ is the premise membership function (rather than $\mu_{premise_{(i)}}$ which was used in our earlier discussion). It is assumed that the functional fuzzy system is defined so that no matter what its inputs are, we have $\sum_{i=1}^{R} \mu_i(z) \neq 0$. The vector z can be chosen in several ways. One common choice is to use $z = [u_1, u_2, \ldots, u_n]^\top$; however, sometimes z might hold other variables, or only a subset of the u_i values (with only a subset of the values, complexity of the mapping generally decreases since the computations needed to find $\mu_i(z)$ are simplified).

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Takagi-Sugeno Fuzzy System

In the special case where

$$b_i = g_i(\cdot) = a_{i,0} + a_{i,1}u_1 + \cdots + a_{i,n}u_n$$

(where the $a_{i,j}$ are fixed real numbers) the functional fuzzy system is referred to as a "Takagi-Sugeno fuzzy system."

$$y = \frac{\sum_{i=1}^{R} b_i \mu_i(z)}{\sum_{i=1}^{R} \mu_i(z)} \qquad (5.3)$$

If $a_{i,0} = 0$, then the $g_i(\cdot)$ mapping is a linear mapping and if $a_{i,0} \neq 0$, then the mapping is called "affine." Often, however, as is standard, we will refer to the affine mapping as a linear mapping for convenience. Overall, we see that the Takagi-Sugeno fuzzy system performs a nonlinear interpolation between linear mappings. In control applications, the linear mappings can each represent a different linear controller and the Takagi-Sugeno fuzzy system interpolates between these and applies combinations of the linear controller outputs (similar in some cases to what is called "gain scheduled control" in conventional control).

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Mathematical Representations

Two Different Approaches

Rules and Membership Functions: To represent linguistic rules, let $\tilde{u}_i$, $i = 1, 2, \ldots, n$, and $\tilde{y}$ denote the linguistic variables that describe u_i, $i = 1, 2, \ldots, n$, and y, respectively. Let $\tilde{A}_i^j$ denote the j^{th} linguistic value for the i^{th} input universe of discourse (here, suppose that $i = 1, 2, \ldots, n$, but that j can, for instance, take on values that are equal to the linguistic-numeric values). Similarly, let $\tilde{B}^p$ denote the p^{th} linguistic value on the output universe of discourse that has linguistic variable $\tilde{y}$. With this, a linguistic rule may be described mathematically by

If $\tilde{u}_1$ is $\tilde{A}_1^j$
and $\tilde{u}_2$ is $\tilde{A}_2^k$
and $\cdots$
and $\tilde{u}_n$ is $\tilde{A}_n^l$
Then $\tilde{y}$ is $\tilde{B}^p$

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Membership Functions

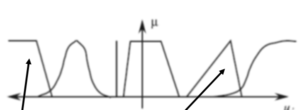

Figure 5.21: Some typical membership functions.

Table 5.4: Mathematical Characterization of Triangular Membership Functions

	Triangular and related membership functions
Left	$\mu^L(u) = \begin{cases} 1 & \text{if } u \le c^L \\ \max\left\{0, 1 + \frac{c^L - u}{0.5w^L}\right\} & \text{otherwise} \end{cases}$
Centers	$\mu^C(u) = \begin{cases} \max\left\{0, 1 + \frac{u - c}{0.5w}\right\} & \text{if } u \le c \\ \max\left\{0, 1 + \frac{c - u}{0.5w}\right\} & \text{otherwise} \end{cases}$
Right	$\mu^R(u) = \begin{cases} \max\left\{0, 1 + \frac{u - c^R}{0.5w^R}\right\} & \text{if } u \le c^R \\ 1 & \text{otherwise} \end{cases}$

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Membership Functions

Table 5.4: Mathematical Characterization of Triangular Membership Functions

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Right	$\mu^R(u) = \begin{cases} \max\left\{0, 1 + \frac{u - c^R}{0.5w^R}\right\} & \text{if } u \le c^R \\ 1 & \text{otherwise} \end{cases}$

A single membership function that represents all three in Table 5.4 is

$$\mu_j^i(u_j) = \begin{cases} 1 & \text{if } (u_j \le c_j^1, i = 1) \text{ or } (u_j \ge c_j^{N_j}, i = N_j) \\ \max\left\{0, 1 + \frac{u_j - c_j^i}{0.5w_j^i}\right\} & \text{if } u_j \le c_j^i \text{ and } (u_j > c_j^1 \text{ and } u_j < c_j^{N_j}) \\ \max\left\{0, 1 + \frac{c_j^i - u_j}{0.5w_j^i}\right\} & \text{if } u_j > c_j^i \text{ and } (u_j > c_j^1 \text{ and } u_j < c_j^{N_j}) \end{cases}$$

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Membership Functions

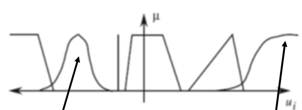

Figure 5.21: Some typical membership functions.

Table 5.5: Mathematical Characterization of Gaussian Membership Functions

	Gaussian and related membership functions
Left	$\mu^L(u) = \begin{cases} 1 & \text{if } u \le c^L \\ \exp\left(-\frac{1}{2}\left(\frac{u - c^L}{\sigma^L}\right)^2\right) & \text{otherwise} \end{cases}$
Centers	$\mu^C(u) = \exp\left(-\frac{1}{2}\left(\frac{u - c}{\sigma}\right)^2\right)$
Right	$\mu^R(u) = \begin{cases} \exp\left(-\frac{1}{2}\left(\frac{u - c^R}{\sigma^R}\right)^2\right) & \text{if } u \le c^R \\ 1 & \text{otherwise} \end{cases}$

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Approach 1

Approach 1: Given Membership Functions, All Possible Rules: Assume that we use center-average defuzzification so that the formula describing how to compute the output is

$$y = \frac{\sum_{i=1}^{R} b_i \mu_i}{\sum_{i=1}^{R} \mu_i} \tag{5.4}$$

where for convenience we use μ_i to represent the premise certainty for the i^{th} rule

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Approach 1

Suppose we use the shorthand notation

$$(j, k, \ldots, l; p)_i$$

to denote the i^{th} rule shown above. In this notation, suppose the indices in (the "tuple") $(j, k, \ldots, l)$ range over $1 \leq j \leq N_1$, $1 \leq k \leq N_2$, ..., $1 \leq l \leq N_n$, and specify which linguistic value is used on each input universe of discourse. Correspondingly, each index in the tuple $(j, k, \ldots, l)$ also specifies the linguistic-numeric value of the input membership function used on each input universe of discourse.

Let

$$b^{(j,k,\ldots,l;p)_i}$$

denote the output membership function (a singleton) center for the i^{th} rule. Note that we use "i" in the notation $(j, k, \ldots, l; p)_i$ simply as a label for each rule (i.e., we number the rules in the rule base from 1 to R, and i is this number). Hence, when we are given i, we know the values of j, k, ..., l, and p. Because of this, an explicit description of the fuzzy system in Equation (5.4) is given by

$$y = \frac{\sum_{i=1}^{R} b^{(j,k,\ldots,l;p)_i} \mu_1^j \mu_2^k \cdots \mu_n^l}{\sum_{i=1}^{R} \mu_1^j \mu_2^k \cdots \mu_n^l} \qquad (5.5)$$

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Approach 1

Approach 1: Given Membership Functions, All Possible Rules: Assume that we use center-average defuzzification so that the formula describing how to compute the output is

$$y = \frac{\sum_{i=1}^{R} b_i \mu_i}{\sum_{i=1}^{R} \mu_i} \qquad (5.4)$$

where for convenience we use μ_i to represent the premise certainty for the i^{th} rule

$$y = \frac{\sum_{i=1}^{R} b^{(j,k,\ldots,l;p)_i} \mu_1^j \mu_2^k \cdots \mu_n^l}{\sum_{i=1}^{R} \mu_1^j \mu_2^k \cdots \mu_n^l} \qquad (5.5)$$

This formula clearly shows the use of the product to represent the premise. Notice that since we use all possible combinations of input membership functions to form the rules there are

$$R = \prod_{j=1}^{n} N_j$$

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Approach 2

Approach 2: Parameterization in Terms of Rules: A different approach to avoiding some of the complications encountered in specifying a fuzzy system mathematically is to use a different notation, and hence a different definition for the fuzzy system. For this alternative approach, for the sake of variety, we will use Gaussian input membership functions. In particular, for simplicity, suppose that for the input universes of discourse we only use membership functions of the "center" Gaussian form shown in Table 5.5. For the i^{th} rule, suppose that the input membership function is

$$\exp\left(-\frac{1}{2}\left(\frac{u_j - c_j^i}{\sigma_j^i}\right)^2\right)$$

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Approach 2

$$\exp\left(-\frac{1}{2}\left(\frac{u_j - c_j^i}{\sigma_j^i}\right)^2\right)$$

for the j^{th} input universe of discourse. Hence, even though we use the same notation for the membership function, these centers c_j^i are different from those used above, both because we are using Gaussian membership functions here, and because the "i" in c_j^i is the index for the rules, not the membership function on the j^{th} input universe of discourse. Similar comments can be made about the σ_j^i, $i = 1, 2, \ldots, R$, $j = 1, 2, \ldots, n$. If we let b_i, $i = 1, 2, \ldots, R$, denote the center of the output membership function for the i^{th} rule, use center-average defuzzification, and product to represent the conjunctions in the premise, then

$$y = \frac{\sum_{i=1}^{R} b_i \prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{u_j - c_j^i}{\sigma_j^i}\right)^2\right)}{\sum_{i=1}^{R} \prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{u_j - c_j^i}{\sigma_j^i}\right)^2\right)} \tag{5.7}$$

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Neural Networks and Fuzzy Systems:

Techniques from one area can be used in the other

In some cases the functionality is identical

MLP: nonlinearity can be tuned by changing weights and biases

Fuzzy Systems: nonlinearity can be tuned by changing the membership functions, incorporate heuristic knowledge

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Multilayer Perceptrons

4.4 Radial Basis Function Neural Networks

A locally tuned overlapping receptive field is found in parts of the cerebral cortex, in the visual cortex, and in other parts of the brain. The radial basis function neural network model is based on these biological systems (but once again, the model is not necessarily accurate, just inspired by its biological counterpart).

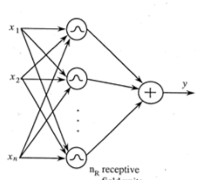

Figure 4.18: Radial basis function neural network model.

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Multilayer Perceptrons

$$y = F_{rbf}(x, \theta) = \sum_{i=1}^{n_R} b_i R_i(x) \tag{4.10}$$

is the output of the radial basis function neural network, and θ holds the b_i parameters and possibly the parameters of the receptive field units.

There are several possible choices for the "receptive field units" $R_i(x)$:

1. We could choose
$$R_i(x) = \exp\left(-\frac{|x - c^i|^2}{(\sigma^i)^2}\right)$$
2. We could choose
$$R_i(x) = \frac{1}{1 + \exp\left(-\frac{|x-c^i|^2}{(\sigma^i)^2}\right)}$$
3. In each of the above cases you can choose to make the σ^i also depend on the input dimension (which makes sense if the input dimensions are scaled differently). In this case for 1 above, for example, we would have $\sigma^i = [\sigma_1^i, \sigma_2^i, \ldots, \sigma_n^i]^\top$ and
$$R_i(x) = \exp\left(-\sum_{j=1}^{n} \frac{(x_j - c_j^i)^2}{(\sigma_j^i)^2}\right)$$
where σ_j^i is the spread for the j^{th} input for the i^{th} receptive field unit. This is the approach that we will use in the example in the next section.

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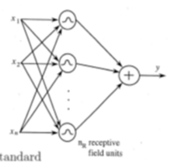
Neural Networks and Fuzzy Systems:

RBF:



Some radial basis function neural networks are *equivalent* to some standard fuzzy systems in the sense that they are functionally equivalent (i.e., given the same inputs, they will produce the same outputs). To see this, suppose that in Equation (4.12) we let $n_R = R$ (i.e., the number of receptive field units equal to the number of rules), let the receptive field unit strengths be equal to the output membership function centers, and choose the receptive field units as

$$R_i(x) = \mu_i(x)$$

(i.e., choose the receptive field units to be the same as the premise membership functions). In this case we see that the radial basis function neural network is *identical* to a certain fuzzy system that uses center-average defuzzification. This fuzzy system is then given by

$$y = F_{rbf}(x, \theta) = F_{fs}(x, \theta) = \frac{\sum_{i=1}^{R} b_i \mu_i(x)}{\sum_{i=1}^{R} \mu_i(x)}$$

where θ holds the membership function parameters for the fuzzy system or strengths and receptive field unit parameters for the radial basis function neural network.

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Tanker Ship Steering

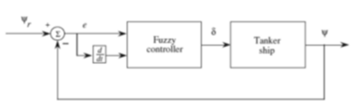

Figure 5.3: Fuzzy controller for a tanker ship steering problem.

Normalization and Scaling:

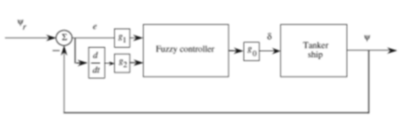

Figure 5.22: Fuzzy controller for tanker ship with scaling gains g_0, g_1, and g_2.

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Normalization and Scaling:

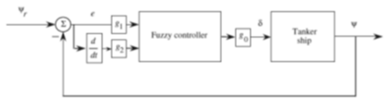

Figure 5.22: Fuzzy controller for tanker ship with scaling gains g_0, g_1, and g_2.

- If $g_1 = 1$, there is no effect on the membership functions and there is no effect on the meaning of the linguistic values.
- If $g_1 < 1$, the membership functions are uniformly "spread out" by a factor of $1/g_1$ (notice that multiplication of each number on the e universe of discourse of Figure 5.23 by π which is $1/g_1$, gives you Figure 5.8 on page 166). This changes the meaning of the linguistics so that, for example, "poslarge" is now characterized by a membership function that represents larger numbers.
- If $g_1 > 1$, the membership functions are uniformly "contracted." This changes the meaning of the linguistics so that, for example, "poslarge" is now characterized by a membership function that represents smaller numbers.

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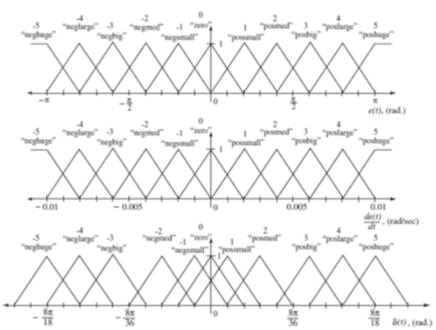

Figure 5.8: Membership functions for a ship steering example.

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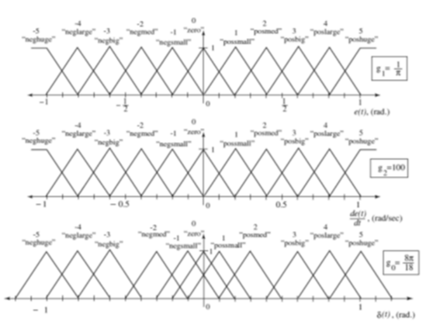

Figure 5.23: Normalized universes of discourse for fuzzy controller for tanker ship (and boxed values of the scaling gains give the original membership functions shown in Figure 5.8).

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Subroutines

- Let `mf1[i]` (`mf2[j]`) denote the value of the membership function associated with input 1 (2) and linguistic-numeric value `i` (`j`). In the computer program, `mf1[i]` could be a subroutine that computes the membership value for the i^{th} membership function given a numeric value for the first input `x1` (note that in the subroutine we can use simple equations for lines to represent triangular membership functions). Similarly for `mf2[j]`.
- Let `rule[i,j]` denote the center of the consequent membership function of the rule that has linguistic-numeric value "`i`" as the first term in its premise and "`j`" as the second term in its premise. Hence `rule[i,j]` is essentially a matrix that holds the body of the rule base table shown in Table 5.2. In particular, for the tanker ship we have `rule[i,j]` as:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0.8 & 0.6 & 0.3 & 0.1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0.8 & 0.6 & 0.3 & 0.1 & 0 & -0.1 \\ 1 & 1 & 1 & 1 & 0.8 & 0.6 & 0.3 & 0.1 & 0 & -0.1 & -0.3 \\ 1 & 1 & 1 & 0.8 & 0.6 & 0.3 & 0.1 & 0 & -0.1 & -0.3 & -0.6 \\ 1 & 1 & 0.8 & 0.6 & 0.3 & 0.1 & 0 & -0.1 & -0.3 & -0.6 & -0.8 \\ 1 & 0.8 & 0.6 & 0.3 & 0.1 & 0 & -0.1 & -0.3 & -0.6 & -0.8 & -1 \\ 0.8 & 0.6 & 0.3 & 0.1 & 0 & -0.1 & -0.3 & -0.6 & -0.8 & -1 & -1 \\ 0.6 & 0.3 & 0.1 & 0 & -0.1 & -0.3 & -0.6 & -0.8 & -1 & -1 & -1 \\ 0.3 & 0.1 & 0 & -0.1 & -0.3 & -0.6 & -0.8 & -1 & -1 & -1 & -1 \\ 0.1 & 0 & -0.1 & -0.3 & -0.6 & -0.8 & -1 & -1 & -1 & -1 & -1 \\ 0 & -0.1 & -0.3 & -0.6 & -0.8 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

(recall that we will scale this matrix of centers by $g_0 = \frac{8\pi}{18}$ after we compute the output of the fuzzy controller).

- Let `prem[i,j]` denote the certainty of the premise of the rule that has linguistic-numeric value "`i`" as the first term in its premise and "`j`" as the second term in its premise given the inputs `x1` and `x2`.
- Let `areaimp[c,h]` denote the area under the output membership function with center `c` that has been chopped off at a height of `h` by the minimum operator. Hence, we can think of `areaimp[c,h]` as a subroutine that is used to compute areas under the membership functions for the implied fuzzy sets.

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Fuzzy controller pseudo code

```
1. Obtain x1 and x2 values
   (Get inputs to fuzzy controller)
2. Compute mf1[i] and mf2[j] for all i, j
   (Find the values of all membership functions given the values
   for x1 and x2)
3. Let num=0, den=0
   (Initialize the COG numerator and denominator values)
4. For i=1 to 11, For j=1 to 11,
   (Cycle through all areas to determine COG)
    prem[i,j]=min[mf1[i],mf2[j]]
    num=num+rule[i,j]*areaimp[rule[i,j],prem[i,j]]
        (Compute numerator for COG)
    den=den+areaimp[rule[i,j],prem[i,j]]
        (Compute denominator for COG)
5. Next i, Next j
6. Output ucrisp=num/den
   (Output the value computed by the fuzzy controller)
7. Go to Step 1.
```

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Fuzzy Controller Tuning for the Tanker Ship

Performance for the First Guess

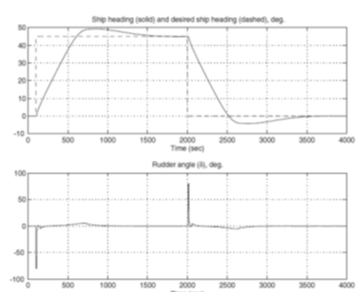

Figure 5.24: Response of fuzzy controller for tanker ship steering, $g_0 = \frac{8\pi}{18}$, $g_1 = \frac{1}{\pi}$, and $g_2 = 100$.

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Tuning the Derivative Gain to Reduce Overshoot

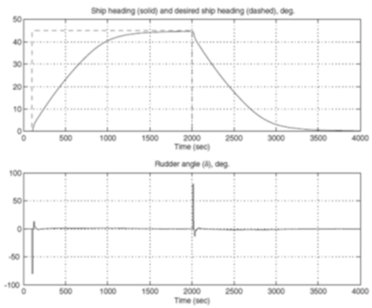

Figure 5.25: Response of fuzzy controller for tanker ship steering, $g_0 = \frac{8\pi}{18}$, $g_1 = \frac{1}{\pi}$, and $g_2 = 200$.

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Tuning the Proportional Gain to Decrease the Response Time

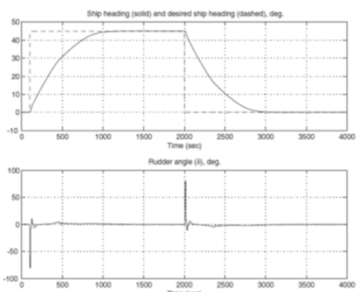

Figure 5.26: Response of fuzzy controller for tanker ship steering, $g_0 = \frac{8\pi}{18}$, $g_1 = \frac{2}{\pi}$, and $g_2 = 250$.

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Nonlinear Control Surface

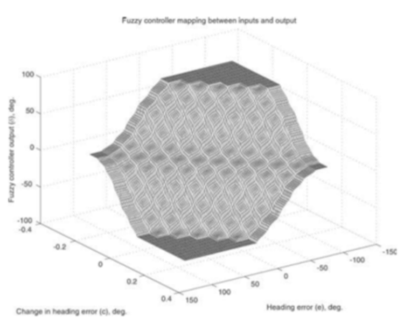

Figure 5.27: Nonlinear control surface implemented by the fuzzy controller,

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Effects of Disturbances and Plant Changes

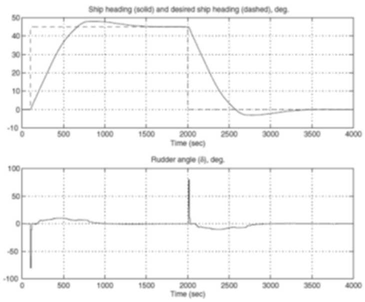

Figure 5.28: Response of fuzzy controller for tanker ship steering, "full" conditions, $g_0 = \frac{8\pi}{18}$, $g_1 = \frac{2}{\pi}$, and $g_2 = 250$.

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Effects of Disturbances and Plant Changes

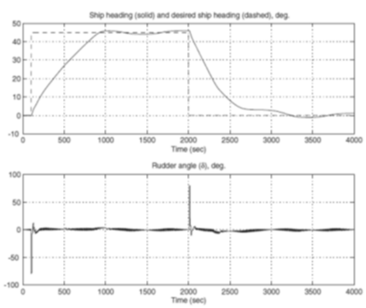

Figure 5.29: Response of fuzzy controller for tanker ship steering, wind disturbance, $g_0 = \frac{8\pi}{18}$, $g_1 = \frac{2}{\pi}$, and $g_2 = 250$.

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Effects of Disturbances and Plant Changes

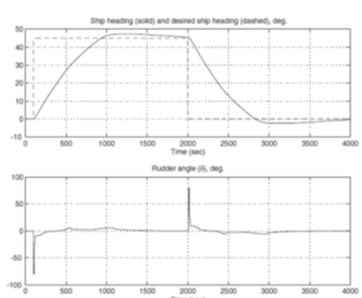

Figure 5.30: Response of fuzzy controller for tanker ship steering, speed decrease, $g_0 = \frac{8\pi}{18}$, $g_1 = \frac{2}{\pi}$, and $g_2 = 250$.

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